Eur. Phys. J. A **3**, 307–309 (1998)

THE EUROPEAN PHYSICAL JOURNAL A
© Springer-Verlag 1998

Short note

$\pi\pi$ S-wave phase shifts and non-perturbative chiral approach

N. Kaiser

Physik Department T39, Technische Universität München, D-85747 Garching, Germany

Received: 17 August 1998 / Revised version: 21 September 1998
Communicated by F. Lenz

Abstract. We extend a recent non-perturbative chiral approach to $\pi\pi$ S-wave scattering of Oller and Oset by including the couplings to the $\eta\eta$-channel. We find that the isospin-zero and isospin-two $\pi\pi$ S-wave phase shifts of the model deviate considerably from a recent solution of the Roy-equations. Including the couplings with the $\eta\eta$-channel does not improve the situation. In particular, no σ-meson like enhancement structure shows up in the Roy-equation solution. We also consider the $\pi\pi$-scattering lengths in this approach.

PACS. 12.39.Fe Chiral lagrangians – 13.75.Lb Meson-meson interactions

1 Introduction and summary

In a recent work, Oller and Oset [1] proposed a non-perturbative approach to deal with the meson-meson interaction in the scalar sector at energies up to $\sqrt{s} \simeq 1.2$ GeV, exploiting chiral symmetry and unitarity in coupled channel. Whereas systematic chiral perturbation theory (renormalized quantum field theory based on the effective chiral Lagrangian) is valid only up to energies $\sqrt{s} \simeq 0.6$ GeV, [1] extended the energy region by going over to a non-perturbative model. In that work the S-wave amplitudes following from the non-linear sigma-model (lowest order chiral Lagrangian) were identified with a coupled channel potential which is iterated to infinite order via a separable Lippmann-Schwinger equation. Using only one adjustable parameter (a cut-off $\Lambda = 1.2$ GeV) a good fit of various empirical meson-meson scattering amplitudes was found. When treating the coupled channels $(\pi\pi, K\bar{K})$ with total isospin $I = 0$, the (narrow) scalar isoscalar $f_0(980)$ resonance is generated as an unstable $K\bar{K}$ bound state. It originates dynamically from the strong $I = 0$ $K\bar{K}$-attraction as predicted by the non-linear chiral Lagrangian. Furthermore, the broad bump in the calculated isospin-zero $\pi\pi$ S-wave phase shift around $\sqrt{s} \simeq 0.6$ GeV was related to a (complex) σ-meson pole at $\sqrt{s} \simeq (0.47 + 0.19\,i)$ GeV. Also, in the coupled channels $(\pi\eta, K\bar{K})$ with isospin $I = 1$, the scalar isovector $a_0(980)$ resonance appears as a quasi-bound $K\bar{K}$-molecule. In the meanwhile the non-perturbative chiral model has been generalized such that even the vector meson poles (ρ, K^*) can be generated via an inverse amplitude method [2] and applications to in-medium modifications are presently studied [3].

The purpose of this short note is to critically reexamine the work of [1]. First, we will include in the description of $I = 0$ $\pi\pi$-scattering the $\eta\eta$-channel whose threshold (only 108 MeV above the $K\bar{K}$-threshold) lies still in the energy region under consideration. Secondly, we will compare the results of the model with a recent solution of the Roy-equations [5,6]. Since Roy-equation solutions fulfill all constraints due to analyticity, unitarity and crossing symmetry they are in general more reliable than plain fits to semi-empirical $\pi\pi$ cross sections as e.g. extracted via extrapolations from $\pi N \to \pi\pi N$ data. As a result we find that there are appreciable discrepancies between the prediction of the chiral coupled channel model (allowing for only one fit parameter λ) and the solution of the Roy-equations. This feature holds in the case of the $I = 0$ $\pi\pi$ S-wave phase shift for the model with and without inclusion of the $\eta\eta$-channel. In particular, the $I = 0$ $\pi\pi$ S-wave phase shift δ_0^0 obtained from solving the Roy-equations rises just monotonically and linearly up to $\sqrt{s} \simeq 0.8$ GeV and it shows no σ-meson like (enhancement) structure. Also, the $I = 2$ $\pi\pi$ S-wave phase shift is not well reproduced above $\sqrt{s} = 0.6$ GeV. We conclude that model of [1] is not able to give an accurate representation of $\pi\pi$ S-wave scattering in the energy region from threshold up to $\sqrt{s} = 1.1$ GeV.

2 Model

In this section we discuss briefly the main ingredients of the model [1] together with the inclusion of the $\eta\eta$-channel. The unique leading order chiral Lagrangian for (pseudoscalar) meson interaction reads,

$$\mathcal{L}^{(2)}_{\phi\phi} = \frac{f^2}{4}\mathrm{tr}\{\partial^\mu U \partial_\mu U^\dagger + \chi(U+U^\dagger)\} \quad (1)$$

with $f = 92.4$ MeV the pion decay constant, $\chi = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$ and the SU(3)-matrix $U = \exp(i\phi\!\!/ /f)$ collecting the octet Goldstone boson fields $(\pi, K, \bar K, \eta)$. We consider the coupled channels $(\pi\pi, K\bar K, \eta\eta)$ in the S-wave with total isospin $I = 0$ and label the corresponding two-meson states by an index j with values (1,2,3), respectively. The S-wave meson-meson scattering amplitudes following from (1) are identified with a coupled channel potential matrix $V_{jk} = V_{kj}$ which reads,

$$V_{11} = \frac{2s - m_\pi^2}{2f^2},\ V_{12} = \frac{\sqrt{3}\,s}{4f^2}\ ,\ V_{13} = \frac{m_\pi^2}{2\sqrt{3}f^2},$$
$$V_{22} = \frac{3s}{4f^2},\ V_{23} = \frac{9s - 8m_K^2}{12f^2},\ V_{33} = \frac{16m_K^2 - 7m_\pi^2}{18f^2}. \quad (2)$$

Note that a positive V_{jj} means here attraction. In order to obtain the coupled channel T-matrix the potential is iterated to infinite order via a separable Lippmann-Schwinger (matrix) equation of the form,

$$T_{jk} = V_{jk} + \sum_{l=1}^{3} V_{jl}\, G_l\, T_{lk}\ , \quad (3)$$

which is easily solved by matrix inversion. Note that extra factors of $1/\sqrt{2}$ and $1/2$ are included in the potentials V_{jk} in order to account for the statistical factor occuring in states with identical particles $(\pi\pi, \eta\eta)$. The quantity G_l is the intermediate state two-meson propagator which derives from the meson-loop with two relativistic scalar propagators, $G_1 = J(s, m_\pi)$, $G_2 = J(s, m_K)$ and $G_3 = J(s, m_\eta)$. Whereas [1] used a three-momentum cut-off to regularize the loop integral $J(s, m)$, we will employ the standard quantum field theoretical result of dimensional regularization and minimal subtraction,

$$J(s,m) = \frac{1}{8\pi^2}\left[\frac{1}{2} - \ln\frac{m}{\lambda} - \sqrt{\frac{4m^2 - s}{s}}\arcsin\frac{\sqrt{s}}{2m}\right],\quad 0 < s < 4m^2,$$
$$J(s,m) = \frac{1}{8\pi^2}\left[\frac{1}{2} - \ln\frac{m}{\lambda} + \sqrt{\frac{s-4m^2}{s}} \times\left(i\,\frac{\pi}{2} - \ln\frac{\sqrt{s}+\sqrt{s-4m^2}}{2m}\right)\right],\quad s > 4m^2. \quad (4)$$

The (renormalization) scale λ plays a role similar to the cut-off in [1] due to the (mild) logarithmic divergence of the meson-loop. Another way to view the differences for G_l is that [1] used an unsubtracted dispersion relation with the imaginary part set to zero above the cut-off Λ, whereas we use a once-subtracted dispersion relation with the subtraction constant given by dimensional regularization and minimal subtraction. As we will see soon, such differences have almost no effect on the numerical results. Finally, one calculates the $I = 0$ $\pi\pi$ S-wave phase shift δ_0^0 and inelasticity η_0^0 from the relation,

$$\eta_0^0\, e^{2i\delta_0^0} = 1 + \frac{i}{8\pi}\sqrt{1 - 4m_\pi^2/s}\; T_{11}\ , \quad (5)$$

where $\eta_0^0 = 1$ up to the $K\bar K$-threshold $s = 4m_K^2$. The $I = 0$ $\pi\pi$ scattering length is given by $a_0^0 = T_{11}/16\pi$, evaluated at $s = 4m_\pi^2$. Within the separable Lippmann-Schwinger equation approach $\pi\pi$ S-wave scattering with total isospin $I = 2$ is a single channel problem,

$$V = \frac{2m_\pi^2 - s}{2f^2},\quad T = \frac{V}{1 - V\, J(s, m_\pi)},$$
$$e^{2i\delta_0^2} = 1 + \frac{i}{8\pi}\sqrt{1 - 4m_\pi^2/s}\; T. \quad (6)$$

3 Results

We use for the meson masses $m_\pi = 139.57$ MeV, $m_K = 493.65$ MeV and $m_\eta = 547.45$ MeV. Let us first consider the model without the $\eta\eta$-channel as done in [1] by setting V_{13} and V_{23} equal to zero. The scale λ is fixed in a best fit of the model to the phase shift δ_0^0 in the region $2m_\pi < \sqrt{s} < 1.1$ GeV obtained from a recent solution of the Roy-equations [5,6]. The optimal value of $\lambda = 1.1$ GeV is very close to the cut-off $\Lambda = 1.2$ GeV of [1]. The resulting phase shift δ_0^0 of the model is shown by the dashed line in the left part of Fig. 1 together with the recent Roy-equation solution (full line). The dashed curve is nearly identical to the one of Oller and Oset [1] which shows that the different treatment of the two-meson propagator G_l has almost no numerical effect. One observes appreciable deviations in Fig. 1. The enhancement of δ_0^0 around $\sqrt{s} = 0.6$ GeV in the model related to the broad σ-meson is not present in the solution of the Roy-equations. Furthermore, the dashed line rises too steeply above $\sqrt{s} = 0.9$ GeV and therefore it ends with a too large value of δ_0^0 at $\sqrt{s} = 1.1$ GeV. We remark aside that the error band associated with earlier Roy-equation solutions [4] has now been substantially reduced due to the precise knowledge of the $I = 0$ $\pi\pi$-scattering length a_0^0 from (two-loop) chiral perturbation theory [7,8].

Next, we consider the complete model with inclusion of the $\eta\eta$-channel. Note that there is a large coupling V_{23} between the $K\bar K$- and $\eta\eta$-channel and one might expect that it will cause a rather different energy dependence of δ_0^0. However, as the right part of Fig. 1 shows, a mere reduction of the scale parameter to $\lambda = 0.9$ GeV (found in a best fit) leads to a result for δ_0^0 which is very similar to the model without the $\eta\eta$-channel. In particular there remain the same deviations from the full curve. The inelasticity η_0^0 above the $K\bar K$-threshold is more sensitive to the presence of couplings to the $\eta\eta$-channel. However, in both cases one does not find an accurate representation of η_0^0 given by the Roy-equation solution [6]. Furthermore, we show in Fig. 2 the isospin $I = 2$ $\pi\pi$ S-wave phase shift.

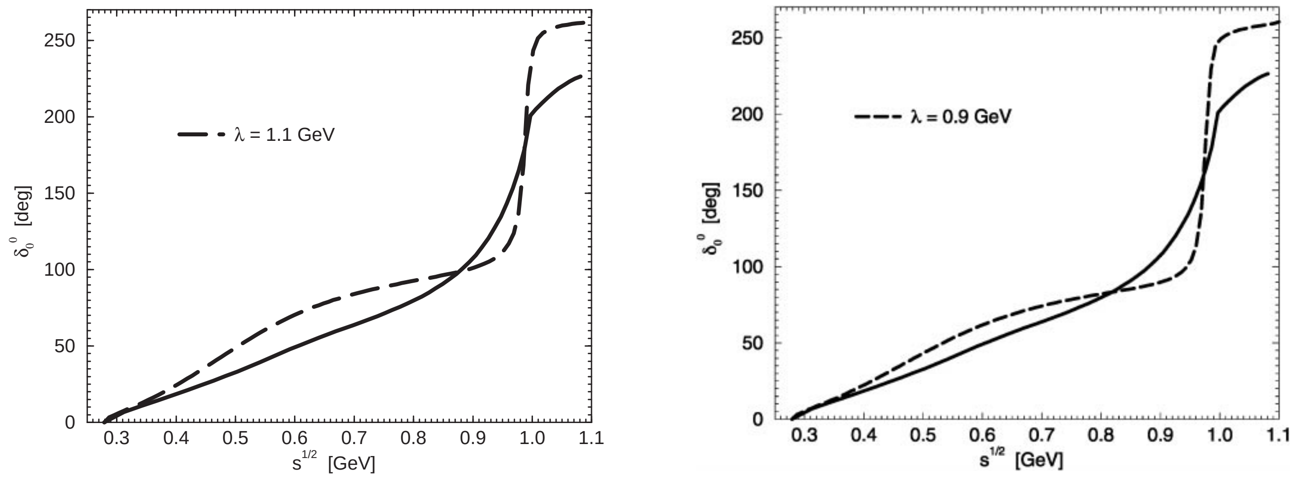

Fig. 1. $I = 0$ $\pi\pi$ S-wave phase shift δ_0^0 versus the center-of-mass energy $\sqrt{s}$. The full lines give the solution of the Roy-equations [6] and the dashed lines correspond to the results of the model. Left: The $\eta\eta$-channel is switched off. Right: The $\eta\eta$-channel is included

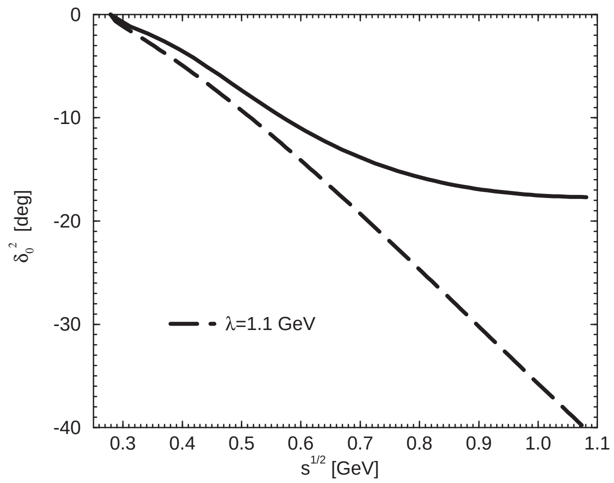

Fig. 2. $I = 2$ $\pi\pi$ S-wave phase shift δ_0^2 versus the center-of-mass energy $\sqrt{s}$. For further notations see Fig. 1

The dashed curve (calculated with $\lambda = 1.1$ GeV) approximately follows the solution of the Roy-equations [6] (full line) up to $\sqrt{s} = 0.6$ GeV. The upward bending at higher energies cannot be reproduced by the simple model (6) using a separable Lippmann-Schwinger equation. We also note that the results for δ_0^2 vary only weakly with the scale λ.

Finally, we mention results for the $\pi\pi$-scattering lengths $a_0^{0,2}$. The model gives $a_0^0 = 0.217$ (with zero couplings to the $\eta\eta$-channel), $a_0^0 = 0.210$ (for the complete model) and $a_0^2 = -0.0423$. These numerical values are close to the results of chiral perturbation theory at one- and two-loop order [7,8]. However, if one considers their pion-mass expansion,

$$\begin{aligned} a_0^0 &= \frac{7m_\pi^2}{32\pi f^2}\left\{1 - 7\left(\frac{m_\pi}{4\pi f}\right)^2 \ln\frac{m_\pi}{\lambda} + \dots\right\}, \\ a_0^2 &= -\frac{m_\pi^2}{16\pi f^2}\left\{1 + 2\left(\frac{m_\pi}{4\pi f}\right)^2 \ln\frac{m_\pi}{\lambda} + \dots\right\}, \end{aligned} \tag{7}$$

one finds that the coefficients of the chiral logarithms are incorrect. They should read 9 and 3 in (7) instead of 7 and 2. Clearly, these differences stem from additional loops in the t- and u-channel and tadpole graphs of chiral perturbation theory which cannot be generated by the Lippmann-Schwinger equation. Numerically, this deficit seems to be cured in the model by (incomplete) higher order terms.

I thank T. Waas for valuable contributions to this work and particularly P. Büttiker for providing me his numerical results of the Roy-equation solutions for $\delta_0^{0,2}$.

References

1. J.A. Oller and E. Oset, *Nucl. Phys.* **A620** (1997) 438; and refs. therein
2. J.A. Oller, E. Oset and J.R. Pelaez, *Phys. Rev. Lett.* **80** (1998) 3452
3. H.C. Chiang, E. Oset and M.J. Vicente-Vacas, preprint nucl-th/9712047.
4. J.L. Basdevant, C.D. Froggatt and J.L. Petersen, *Nucl. Phys.* **B72** (1974) 413
5. B. Ananthanarayan and P. Büttiker, *Phys. Rev.* **D54** (1996) 1125, 5501; and refs. therein
6. P. Büttiker, PhD Thesis, Universität Bern, 1996; and private communication
7. J. Gasser and H. Leutwyler, *Phys. Lett.* **B125** (1983) 321, 325
8. J. Bijnens *et al.*, *Phys. Lett.* **B374** (1996) 210